WEDGE AND CONE THEORY FOR $M_\infty = \infty$

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Nomenclature

C_p	surface pressure coefficient, $\frac{p - p_\infty}{q_\infty}$
M	Mach number
p	pressure
q	dynamic pressure
u, v	velocity components parallel and perpendicular to free stream
V	velocity
γ	ratio of specific heats
δ	cone or wedge angle
θ	shock angle

Subscripts

∞	free-stream conditions
c	cone
D	detachment
e	effective value
w	wedge
2	conditions immediately behind oblique shock

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The tabulated values of the ratio of cone surface pressure to pressure at the shock, p_c/p_2, presented in Ref. 1 for $\gamma = 1.405$ and 1.333 at $M_\infty = \infty$ indicate that this ratio is sensibly independent of cone angle for a range of cone angles from 5° to 50°. These tabulated values are shown in Fig. 1 together with solutions for $\gamma = 1.667$ from an unpublished Ames cone program.

It is evident that the exact solution is independent of cone angle for all γ when $M_\infty = \infty$ and is, therefore, just a function of γ. This dependence on γ can be closely approximated by Eq. (1) as demonstrated in Fig. 1.

$$\frac{p_c}{p_2} = \left[\frac{4(\gamma + 1)^2}{(\gamma^{1/2} + 1)^4}\right]^{\gamma/\gamma-1} \tag{1}$$

Since for wedge flows, p_w/p_2 is also independent of wedge angle, namely $p_w/p_2 = 1$, a relationship between wedge and cone flows at $M_\infty = \infty$ is indicated.

To investigate this relationship in more detail, it is advantageous to examine the flows over wedges and cones in the hodograph plane (u, v coordinates). As shown in Fig. 2, the hodographs of wedge flows are circles of radius $\gamma/\gamma + 1$ with centers on the abscissa at $\gamma/\gamma + 1$, while the hodographs of cone flows are the Busemann "apple" curves. By analogy to wedge flows, it is of interest to note how closely the "apple" curve is approximated by an arc of a circle for cone angles up to detachment. The dimensions of this approximating circle can be computed from an effective specific-heat ratio that is obtained from known detachment conditions of Ref. 1 and from wedge theory for detachment, where $\sin \delta_{D_w} = 1/\gamma$.

[redacted]

Reference 1 gives $\delta_{D_c} = 57.525^\circ$ for $\gamma = 1.405$ from which an effective specific-heat ratio $\gamma_e = 1/\sin 57.525$ can be estimated. However, an accurate effective specific-heat ratio can be obtained if $\gamma_e = \gamma^{1/2}$. If this approach is used for a specific-heat ratio of 1.4050, the effective specific-heat ratio is 1.1853 for which the cone angle for detachment would be 57.527°. This value differs from that previously cited from Ref. 1 in the fifth significant figure. By analogy, the circular arc that approximates the conical flow hodograph has a radius of $1/(\gamma^{1/2} + 1)$ with center at $\gamma^{1/2}/(\gamma^{1/2} + 1)$ as shown in Fig. 2. Within the accuracy of Fig. 2, the approximating circle cannot be distinguished from the exact numerical solution[1,2] for all cone angles up to detachment.

The approximating equation for shock angle of conical flow is derived from wedge theory by replacing γ by $\gamma^{1/2}$. The exact solution for weak shock angles of wedge flows from ref. 3 is

$$\sin^2 \theta = \frac{1}{2}\left[1 + \gamma \sin^2 \delta_w - \cos \delta_w(1 - \gamma^2 \sin^2 \delta_w)^{1/2}\right] \qquad (2)$$

and, therefore, for conical flow

$$\sin^2 \theta = \frac{1}{2}\left[1 + \gamma^{1/2} \sin^2 \delta_c - \cos \delta_c(1 - \gamma \sin^2 \delta_c)^{1/2}\right] \qquad (3)$$

Similarly, the shock angle for detachment of wedges is given by

$$\sin^2 \theta_{D_w} = \frac{\gamma + 1}{2\gamma} \qquad (4)$$

and for cones by

$$\sin^2 \theta_{D_c} = \frac{\gamma^{1/2} + 1}{2\gamma^{1/2}} \qquad (5)$$

Equations (2) and (3) and exact solutions of shock angle for conical flow are compared in Fig. 3. It is believed that Eqs. (3) and (5) give shock angles for cones with an error less than 0.1° for $\gamma = 1.400$. It is of interest to note from Fig. 3 that the exact detachment curve for wedge flow closely approximates that for cone flow.

The pressure coefficient for conical flow at $M_{\infty} = \infty$ can be expressed as

$$C_p = \frac{p_c}{p_2}\left(\frac{4}{\gamma + 1}\right) \sin^2 \theta \tag{6}$$

It follows from Eqs. (1), (3), and (6) that a close approximation for pressure coefficient as a function of cone angle is given by

$$C_p = \left(\frac{2}{\gamma+1}\right) \left[\frac{4(\gamma+1)^2}{(\gamma^{1/2}+1)^4}\right]^{\gamma/\gamma-1} \left[1+\gamma^{1/2} \sin^2 \delta_c - \cos \delta_c (1-\gamma \sin^2 \delta_c)^{1/2}\right] \tag{7}$$

The velocity at the cone surface can be obtained from the hodograph of the cone in Fig. 2 as

$$\frac{V_c}{V_{\infty}} = \frac{\gamma^{1/2}}{\gamma^{1/2} + 1} \cos \delta_c + \frac{1}{\gamma^{1/2} + 1} (1 - \gamma \sin^2 \delta_c)^{1/2} \tag{8}$$

References

1. Mass. Inst. of Tech., Dept. of Elect. Engr, Center of Analysis: Tables of Supersonic Flow Around Cones by the Staff of the Computing Section Under the Direction of Zdeněk Kopal. Tech. Rep. 1, Cambridge, 1947.

2. Hantzsche, W., and Wendt, H.: Cones in Supersonic Flow. NACA TM 1157, 1947.

3. Cleary, J. W., and Axelson, J. A.: Theoretical Aerodynamic Characteristics of Sharp and Circularly Blunt-Wedge Airfoils. NASA TR R-202, 1964.

4. Ames Research Staff: Equations, Tables, and Charts for Compressible Flow. NACA Rep. 1135, 1953.

Figure Titles

Fig. 1 Ratio of cone surface pressure to the pressure just behind the shock; $M_\infty = \infty$.

Fig. 2 Hodographs of wedge and cone flows for $M_\infty = \infty$ and $\gamma = 1.405$.

Fig. 3 Comparison of shock wave angles for wedges and cones; $M_\infty = \infty$.

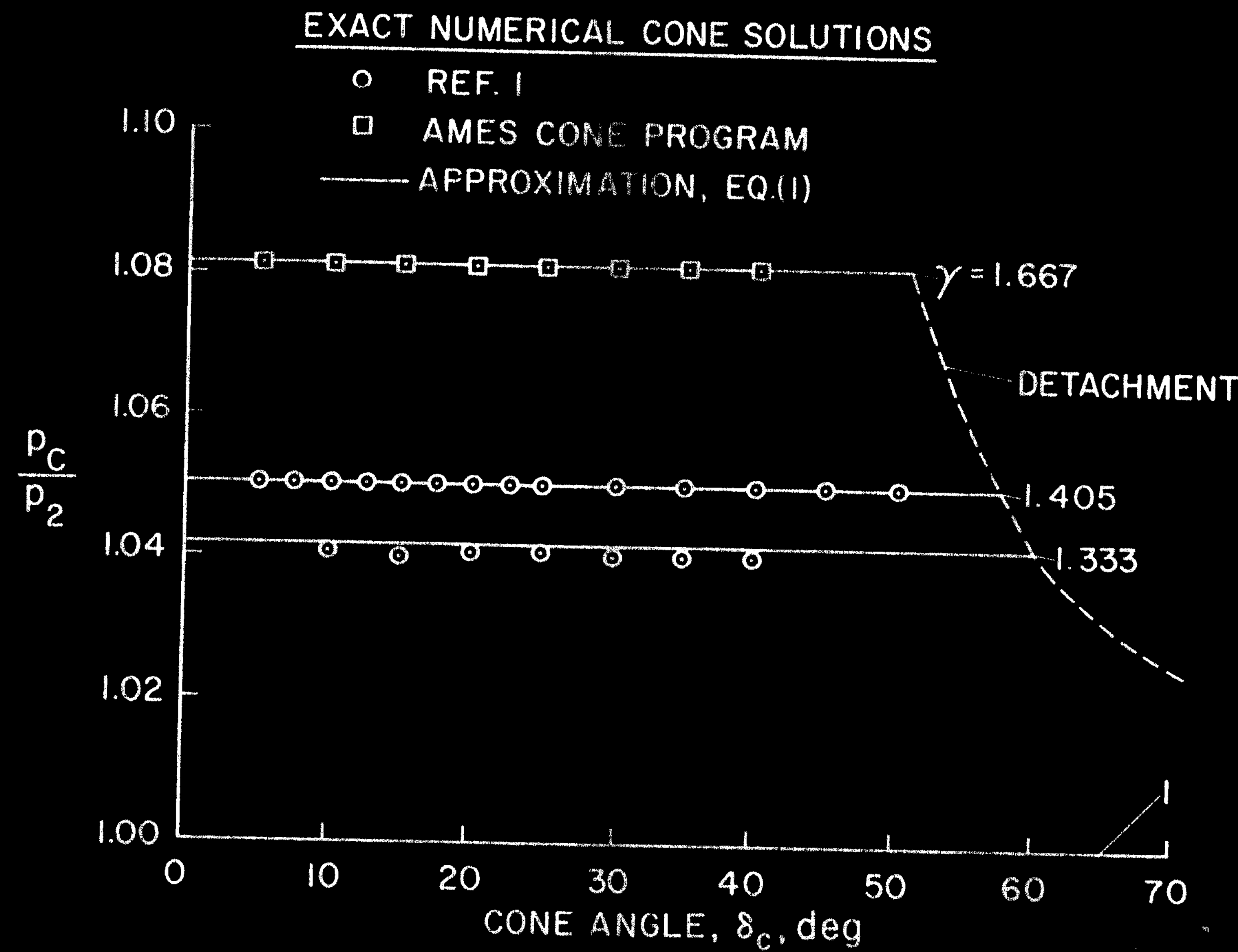
EXACT NUMERICAL CONE SOLUTIONS
REF. I
AMES CONE PROGRAM
APPROXIMATION, EQ.(I)
$\gamma = 1.667$
DETACHMENT
1.405
1.333
1
1.10
1.08
1.06
1.04
1.02
1.00
$\frac{p_c}{p_2}$
0
10
20
30
40
50
60
70
CONE ANGLE, δ_c, deg

Fig. 1

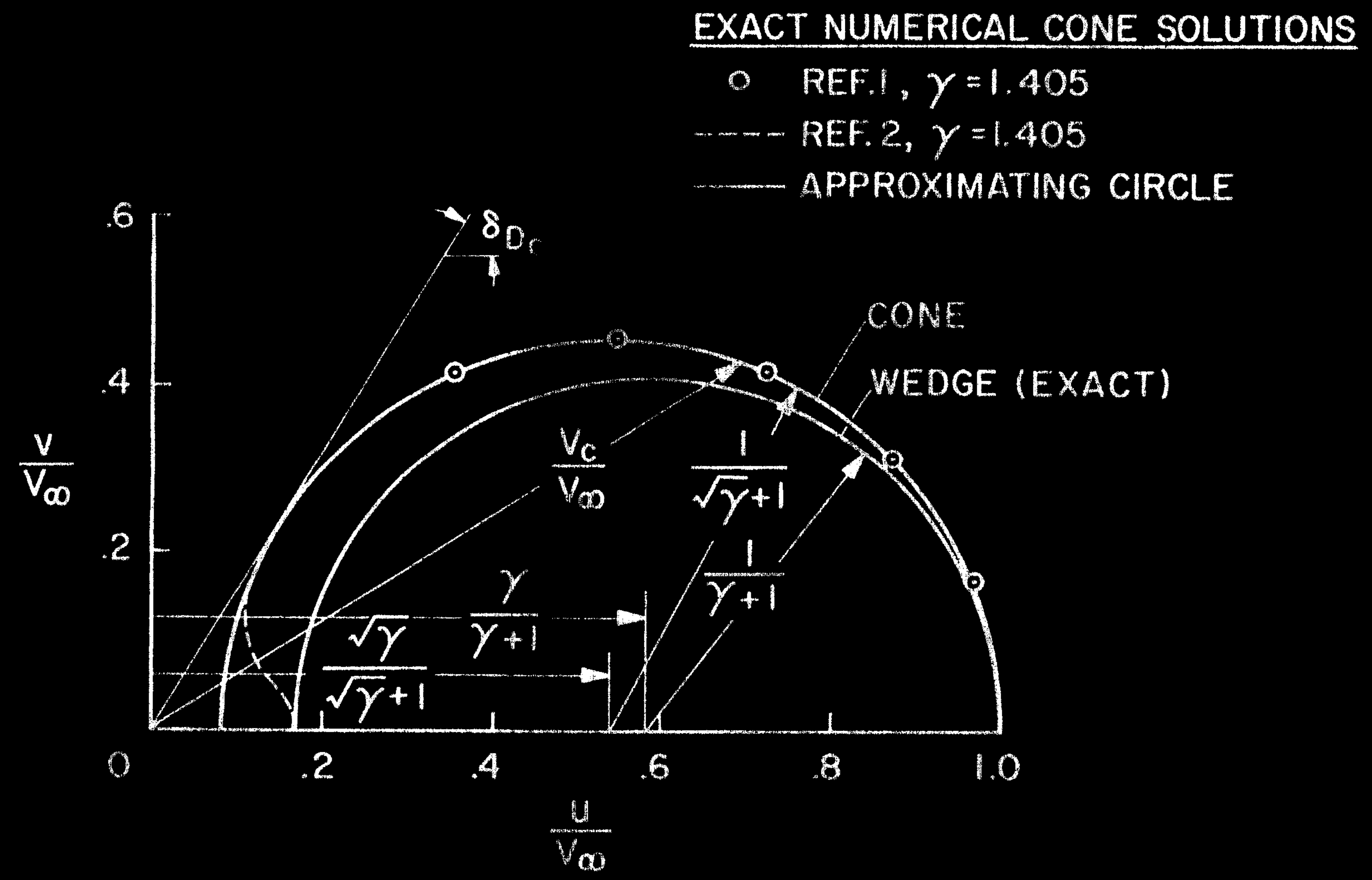

Fig. 2

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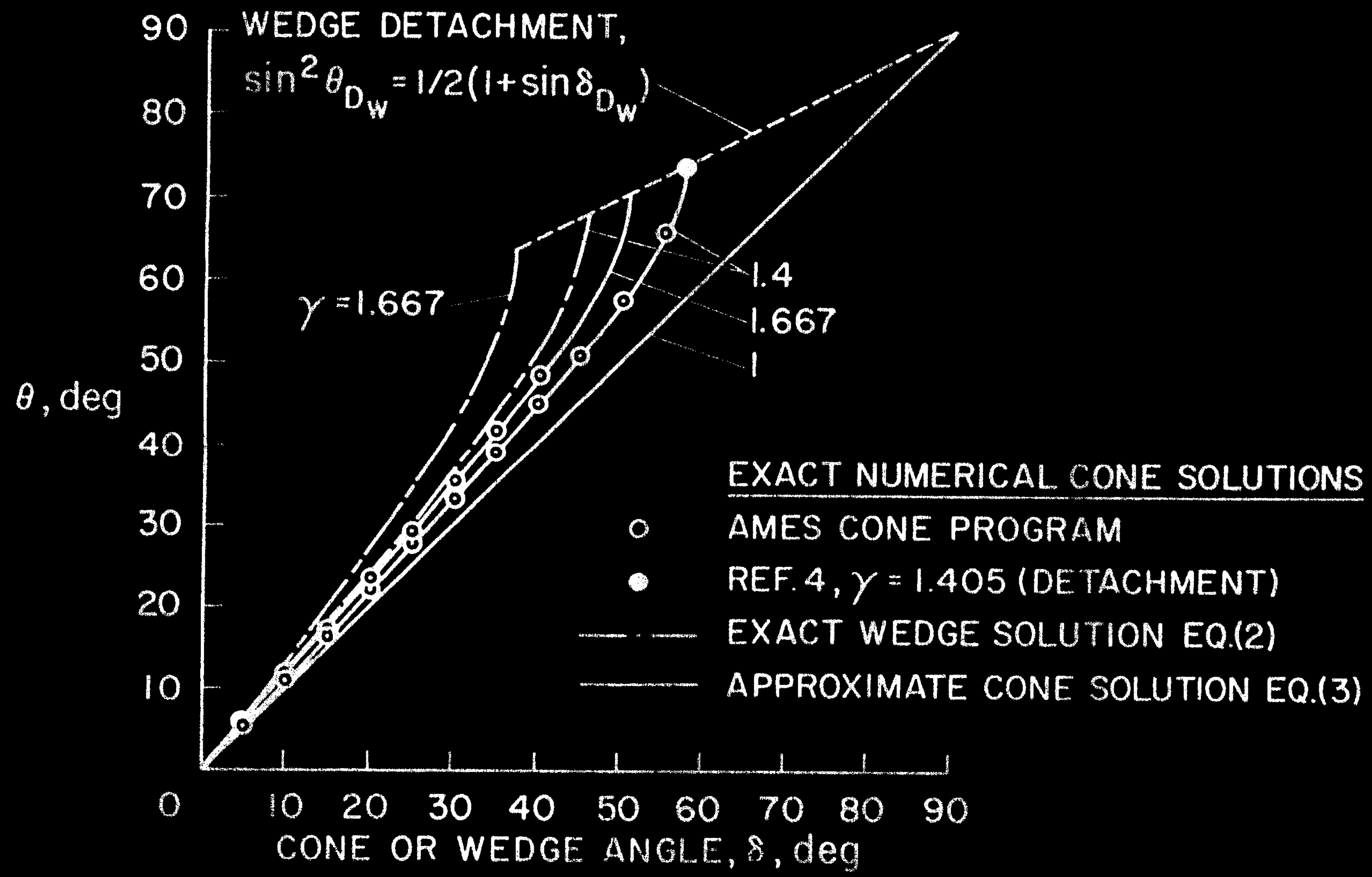
WEDGE DETACHMENT,
$\sin^2\theta_{D_w} = 1/2(1+\sin\delta_{D_w})$
$\gamma = 1.667$
1.4
1.667
1
θ, deg
0
10
20
30
40
50
60
70
80
90
CONE OR WEDGE ANGLE, δ, deg
EXACT NUMERICAL CONE SOLUTIONS
AMES CONE PROGRAM
REF. 4, γ = 1.405 (DETACHMENT)
EXACT WEDGE SOLUTION EQ.(2)
APPROXIMATE CONE SOLUTION EQ.(3)

Fig. 3